Multiple Dielectric Slabs in Waveguide Cell

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***Abstract*—The transverse resonance method is used to evaluate the effective dielectric constant of a material present in the form of multiple slabs that partially fill a waveguide cell. It is shown that the commonly-used linear relationship for the effective dielectric constant of a mixture of dielectrics holds good to better than 1% accuracy only when the slabs are, in general, of a width each less than 5% of the wavelength in the dielectric. Alternative mixture equations with empirical terms are also presented.**

***Index Terms*—Dielectric constant, layered dielectrics, transverse resonance, wave guide cell.**

I. INTRODUCTION

TECHNIQUES for the measurement of the dielectric constant of a homogeneous material completely filling a waveguide cell are well developed. However, there exists considerable interest in the measurement of the dielectric constant of inhomogeneous materials, and materials which only exist as a mixture of two or more components, such as granular materials with air voids, layers of fat and meat in animal products, and also unevenly shaped materials that do not completely fill the waveguide cell leaving an air void. The usual practice in such cases is to measure the permittivity of the mixture and use a linear mixture equation to deduce the dielectric constant of the material [1], namely

$$\varepsilon_{\text{eff}} = v\varepsilon_d + (1-v)\varepsilon_0 \tag{1}$$

where v is the volume fraction of the dielectric material, ε_{eff} is the effective dielectric constant of the mixture, and ε_d and ε_0 are the dielectric constants of the material and air, respectively. Several experiments were reported in [2] substantiating (1).

The motivation for this work is to investigate the validity of (1) for the multiple slab loaded waveguide cell. In particular, to consider the case shown in Fig. 1 where a loss-less material is in the form of a number of longitudinal slabs interspersed with air gaps, and to determine the upper value of the electrical width of the material slabs for which (1) is valid to within some specified accuracy. This investigation is undertaken by computing the cutoff frequency of the slab-loaded waveguide using the transverse resonance method (TRM).

Manuscript received September 18, 2000; revised December 21, 2000. This work was supported by New Zealand Foundation for Research Science and Technology, under Contract C08806, Objective 4.

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Publisher Item Identifier S 1531-1309(01)03215-9.

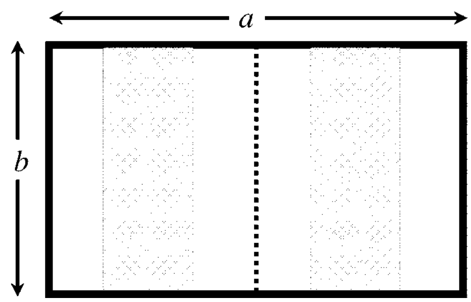

Fig. 1. Multiple slab loaded waveguide for $M = 2$.

II. THEORY

Consider a metal walled rectangular waveguide of dimensions a and b in the x and y directions, respectively. If the cutoff frequency of the waveguide when filled with a loss-less homogeneous dielectric, or a mixture of loss-less dielectrics, is $f_c(\varepsilon_{\text{eff}})$ and that of an air-filled waveguide is $f_c(\varepsilon_0)$, then

$$\frac{\varepsilon_{\text{eff}}}{\varepsilon_0} = \left[\frac{f_c(\varepsilon_0)}{f_c(\varepsilon_{\text{eff}})}\right]^2. \tag{2}$$

Consequently the measurement, or calculation, of the cutoff frequency of a waveguide filled with a mixture of dielectrics permits the effective dielectric constant to be determined by (2).

Consider now the situation of a slab-loaded waveguide where the volume fraction of the loss-less dielectric in the waveguide cell is v. In particular, we consider the case where the dielectric is present as M parallel slabs aligned in the longitudinal direction, each slab of dielectric (of thickness $t_d = av/M$) surrounded on both sides by layers of air of thickness $a(1-v)/2M$, with this pattern (of air-dielectric-air) being repeated M times across the width of the waveguide. The case for $M = 2$ is shown in Fig. 1. This arrangement provides symmetric loading across the cell, and as M increases the electrical thickness of each slab decreases but the ratio of dielectric to air in the cell remains constant. This slab loaded waveguide can be considered as $3M$ regions across the waveguide width where the dielectric constant, ε_i, and thickness, t_i, of the ith slab are given by the following for $p = 1, 2 \ldots M$:

$$\begin{array}{ll} \varepsilon_i = \varepsilon_d, & \text{for } i = (3p-1), \text{ and} \\ \varepsilon_i = \varepsilon_0, & \text{for } i = 3p \text{ and } i = (3p-2); \text{ and} \\ t_i = t_d = av/M, & \text{for } i = (3p-1), \text{ and} \\ t_i = a(1-v)/2M, & \text{for } i = 3p \text{ and } i = (3p-2). \end{array} \tag{3}$$

This problem can now be analyzed by the transverse resonance method (TRM) [3]–[5].

III. RESULTS AND DISCUSSION

Consider a rectangular waveguide with dimensions with $a = 72.136$ mm and $b = 34.036$ mm within which there are M

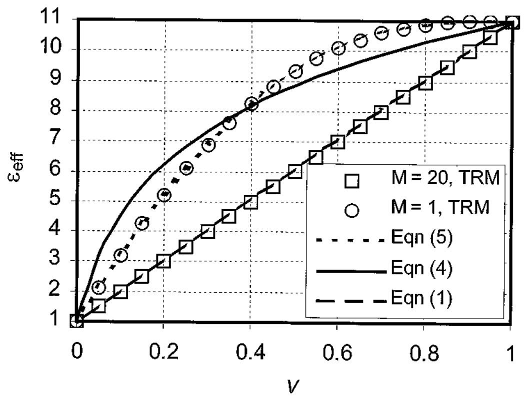

Fig. 2. Computed values of effective dielectric constant as a function of number of dielectric layers for different values of v.

slabs of a dielectric with $\varepsilon_d = 11$. Using the cutoff frequency determined by TRM we can deduce ε_{eff} from (2) which is shown in Fig. 2 as a function of M for various values of v. Note that ε_{eff} depends on M for small M, but that as M increases, and therefore the electrical width of the slabs decreases, ε_{eff} tends to a constant value which can be shown to be that predicted by (1). Fig. 3 shows the surface of relative error between the TRM result and that obtained from (1), as a function of the volume fraction v and M. As expected, (1) is correct in the two extremes $v = 0$ and $v = 1$. Note that the error surface is convex with respect to v and the error is larger when v is in the range of 0.2 to 0.3, approximately.

Calculations have shown that the situation is similar for other values of ε_d. It can be seen from Figs. 2 and 3 that the value of ε_{eff} predicted by (1) is always smaller than that determined from (2) using TRM, and the difference between the results decreases as the value of M increases, that is as the electrical thickness of the dielectric slabs decreases. Values of ε_{eff} obtained from (2) for $M = 1$ and $M = 20$ are shown in Fig. 4, which indicates that for large values of M, ε_{eff} is almost linear with respect to v as predicted by (1).

For the same size waveguide, Fig. 5 shows the percentage error in ε_{eff} predicted by (1) compared to that determined by TRM and (2) as a function of the electrical thickness of each slab for three values of $\varepsilon_d = 3,\ 8$, and 11. Note that for (1) to predict the effective dielectric constant with an error of less than 1%, the ratio of the thickness of the dielectric (t_d) to the wavelength in the dielectric (ε_d) must be less than 0.075 for $\varepsilon_d = 3$, decreasing to 0.05 for $\varepsilon_d = 11$.

The nonlinear behavior, as a function of v, of the effective dielectric constant of a mixture of two dielectrics (as seen in Figs. 3 and 4) can be better modeled by a mixture equation of the form [6]

$$e^{\alpha\varepsilon_{\text{eff}}} = ve^{\alpha\varepsilon_d} + (1 - v)e^{\alpha\varepsilon_0} \tag{4}$$

where α is selected on some best fit criteria. Note that (4) reduces to (1) as α tends to zero. An alternative approach is to incorporate an additional empirical error term $d\varepsilon_{\text{eff}} = q.\sin(\pi v^w)$ in (1) to obtain

$$\varepsilon_{\text{eff}} = v\varepsilon_d + (1 - v)\varepsilon_0 + q\sin(\pi v^w) \tag{5}$$

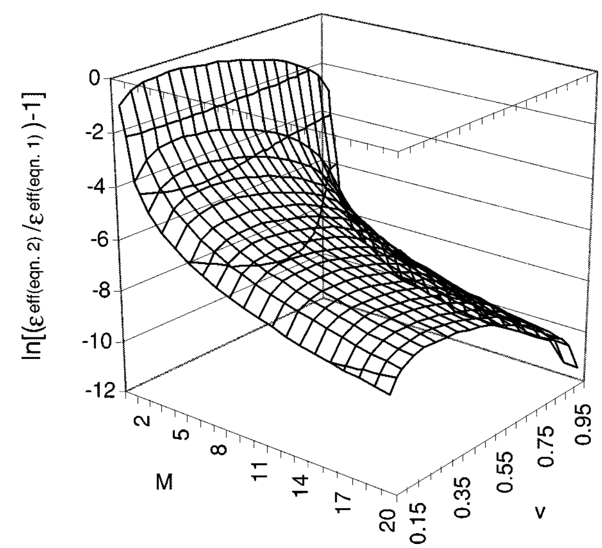

Fig. 3. Relative error in computed values of effective dielectric constant with respect to that obtained from (1) for the slab loaded waveguide as a function of volume fraction of material of slab v for various M and $\varepsilon_r = 11$.

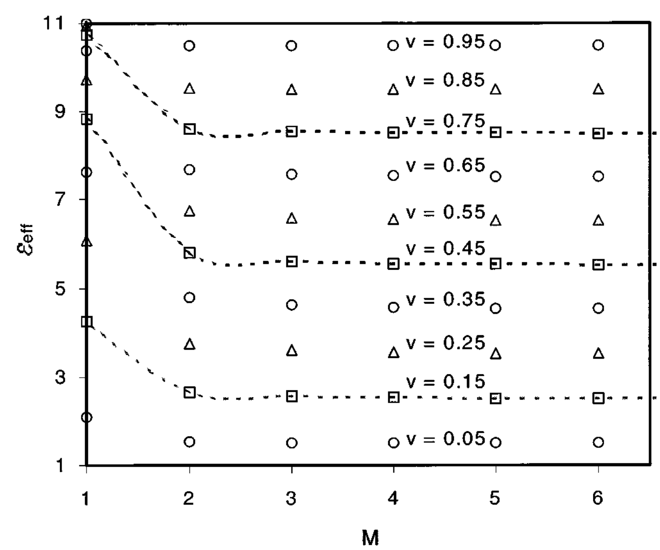

Fig. 4. Values of effective dielectric constant of mixture, ε_{eff} as a function of volume fraction of material of dielectric slab v for $M = 1$ and $M = 20$.

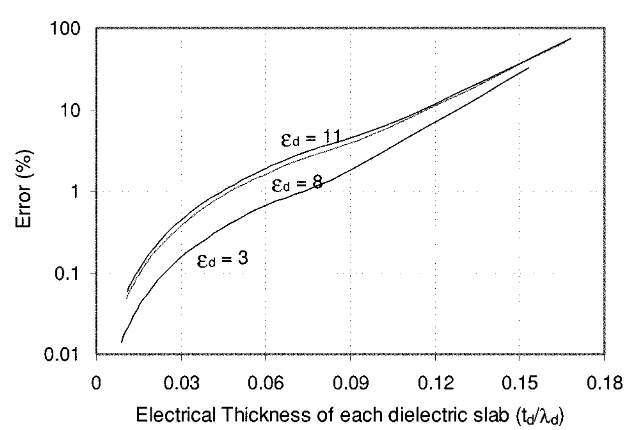

Fig. 5. Percentage error in ε_{eff} predicted by (1) as a function of electrical thickness of each dielectric slab t_d/λ_d for $v = 0.25$ and $\varepsilon_d = 3$, 8, and 11.

where both q and w are selected on some best fit criteria. In Fig. 4, results are shown for $\varepsilon_{\mathrm{eff}}$ for $M = 1$ predicted by (4) with $\alpha = 0.3$ and by (5) with $q = 3.4$ and $w = 0.93$. It can be seen that (4) is a better predictor of $\varepsilon_{\mathrm{eff}}$ than (1), and (5) is better still.

IV. Conclusion

For dielectric constants up to about 11 it would be necessary for the slab thickness to be less than 5% of the wavelength in the dielectric in order for the linear mixture equation (1), to predict the effective dielectric constant with an error of less than 1%. To achieve an error of less than 0.1% the thickness would have to be reduced to less than 1.4% of the wavelength in the dielectric. Generalization of these results to lossy dielectrics as well as further research using inverse numerical techniques is in progress.

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